A Synthesis of the Theory of Corporate Finance

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ABSTRACT

This paper synthesizes the theory of corporate finance. It highlights the equations, graphs and models of corporate finance theory with particular reference to capital markets, consumption and investments, investment decision under certainty and uncertainty assumptions, capital budgeting decision under certainty, uncertainty and capital rationing Conditions. There is also a down to earth explanations of the theory of choice, objects of choice, market equilibrium, the theory of efficient capital markets, theories of dividend policy, capital structure and cost of capital, security valuation, contingent claims analysis and agency theory and analysis. This synthesis modeled eighty (80) equations and twenty (20) graphs as it relates to corporate finance theory.

Keywords: Corporate Finance Theory, Equations, Graph and Model

Introduction

Over the years, the field of finance has remained dynamic and has received considerable attention from academic researchers keen on understanding issues likes:

- How do individuals and society allocate scare resources?
- How do people make choices?
- How should capital budgeting decision be made?
- What dividend policy should the firm follow?
- How efficiently does capital market function?
- What is the cost of capital? How is it affected by project risk? and by project debt capacity?
- How should financial assets be valued?

It is on this basis this synthesis is carried out to proffer answers to this questions thereby reviewing some equations, graphs, theories and models relating to corporate finance and it implication for corporate survival.
1. Capital Markets, Consumption and Investment

A. Consumption and Investment without Capital Markets

Assumption

- All outcomes from investment are known with certainty
- No transaction costs
- No taxes
- Two-period model

![Figure 1: How Individuals Make Choices among Consumption Bundles](image)

At point A, the decision maker has more consumption at end of the period but less consumption at the beginning than point B decision maker does. The straight line tangent at point B measures the rate of trade-off between C1 and C0. The trade-off is called the marginal rate of substitution (MRS). It also reveals the individual subjective rate of time preference r1 at point B. mathematically, it is defined as:

\[ MRS_{C1} = \frac{\partial C1}{\partial C0} \bigg|_{u=\text{const}} = (1+r_1) \]

Where \( C_1 \) = Consumption at end of the period
$C_0$ = Consumption at the beginning of the period

**Fig. 2 Investment Schedule and Production Opportunity Set**

The above shows a situation where productive opportunities abound, which allows a unit of current savings/investment to be turned into more than one unit of future consumption. At any given point an individual will make all investments which have rate of return higher than his subjective rate of time preference.
Fig. 3 The Investment Opportunity Set

Note: The slope of a line tangent to curve ABC in Fig.3 demonstrate the rate at which a naira of consumption forgone today is transformed into a naira of consumption tomorrow it is the marginal rate of transformation offered by the investment opportunity set.

Fig. 4 Equilibrium in a Robinson Crusoe World

Individual 1 consumes more than individual 2 in period one since no capital market, individuals with the same endowment and the same investment opportunity set may choose completely differed investments because they have different indifference curve.

Consumption and Investment with Capital Market

Initial endowment A:

\[ W_0 = Y_0 + \frac{Y_1}{1 + r} \]  

(2)

Where \( W_0 \) = the present value of an individual initial endowment
Y_0 = current income
Y_1 (1+r) = the present value of his end of period income.
Fig. 5 Capital market line

With capital market, an individual can move from A to B, reaching a higher utility. On the capital market line, one’s wealth does not change but consumption differs. The slope of the capital market line is given as:

\[ W_0 = C_0^* + \frac{C_1^*}{1 + r} \]

Equation 3 can be rewritten to give the CML equation as

\[ C_1^* = W_0 (1 + r) - (1 + r) C_0^* \]

and since \( W_0 (1+r) = W_1 \), we have

\[ C_1^* = W_1 - (1 + r) C_0^* \]

\( w_1 \) is the intercept, while \((1 + r)\) is the slope of the capital market line.
Both POF and CML offer a high rate of return than his subjection time preference, but production offers higher return, therefore, an agent chooses to invest and move along the production frontier.

**FISHER SEPARATION THEORY**

“Given perfect and complete markets, the production is governed by an objective market criterion without regard to individual’s subjective preferences that enter into their consumption decision”. The separation of investment and consumption decisions is known as the **Fisher Separation Theorem**.
Fig. 7 Investment Decision is Independent of Individual Preferences

Without capital market opportunities, individual 1 would choose to produce at point Y, which has lower utility, similarly, individual 2 would be worse off at point X. But with good investment opportunities which rates of returns are higher the market than rate, the individual can borrow (in case of wealth insufficient) funds and invest more than they would without capital markets.
2. Investment Decisions

A. Under the Certainty Assumption:

The following assumptions are made:

- The interest rate is known in all time periods
- All future payoffs from current investment decision are known
- There are no imperfections to capital markets.

![Diagram](image)

**Fig. 8 Separation of Shareholder Preferences from the Production Investment Decision**

In the graph, \( P_0, P_1 \) maximizes the present value of the shareholders wealth \( W_0 \). The separation principle implies that the maximization of the shareholders wealth is identical to maximizing the present value of their life time consumption. Mathematically, this is demonstrated as

\[
W_0 = C_0 + \frac{C_1}{1+r} \tag{6}
\]
Where $W_0 =$ current wealth
$C_0 =$ current consumption
$C_1 =$ end of period consumption (future)

**B. Maximization of Shareholders Wealth**

We can say that shareholders wealth is the discounted value of after tax cash flows paid out by the firm which can be shown to be same as stream of dividends $D_t$, paid to shareholder. The discounted value of the stream of dividend is

$$S_0 = \sum_{t=0}^{\infty} \frac{D_t}{(1+k)^t}$$  \hspace{1cm} (7)

Where $S_0 =$ is the present value of shareholder wealth
$K =$ is the market determined rate of interest.

Equation 7 can be rewritten for the present value of a growing annuity stream, we get

$$S_0 = \frac{D_1}{k - g}$$  \hspace{1cm} (8)

Where $g =$ is the growth rate of the dividend stream and
$K =$ the opportunity cost of investment.

Equation 7 and 8 is the discounted value of the stream of cash payments to shareholders and is equivalent to the shareholders wealth.

**C. Definition of Profit**

To an economist profits mean cash flow. In making decision, the profits to be used are the discounted stream of cash flows to shareholders (dividend) =

$$D_t = R_t - (W & S)_t - I_t$$  \hspace{1cm} (9)

We can rewrite shareholders wealth as

$$S = \sum_{t=0}^{\infty} \frac{R_t - (W & S)_t - I_t}{(1+k)^t}$$  \hspace{1cm} (10)

The accounting definition of profit is mathematically express as

$$A_t = R_t - (W & S)_t - z_t$$  \hspace{1cm} (11)
Since $A_t$ is different from $D_t$, it can be adjusted by subtracting net investment. This is done in eq (12)

$$
S = \sum_{t=0}^{\infty} \frac{R_t - (W \& S)_t - Z_t - (I_t - Z_t)}{(1 + k)^t}
$$

$$
= \sum_{t=0}^{\infty} \frac{A_t - N_t}{(1 + k)^t}
$$

(12)

D. **Techniques for capital budgeting**

The best capital budgeting techniques possess the following essential property.

- All cash flows should be considered
- The cash flows should be discounted at the opportunity cost of funds.
- The technique should select from a set of mutually exclusive projects the one which maximizes shareholders wealth.
- Managers should be able to consider one project independently from all other (This is known as **The Value Additivity Principle**)

The VAP implies that if we know the value of separate projects accepted by management, then simply adding their values $V_j$ will give us the value of the firm $V$.

$$
V = \sum_{j=1}^{N} V_j
$$

(13)

- There are five (5) widely used capital budgeting technique

1. **The payback method**: This method is defined as follows

$$
\text{Payback} = \frac{\text{Initial Investment}}{\text{Annual cash inflow}} = \frac{C_0}{C}
$$

(14)

2. **The accounting rate of return (ARR)**

$$
\text{ARR} = \frac{\text{Average after-tax income}}{\text{Initial outlay}}
$$

or

$$
\text{ARR} = \left[ \frac{\sum_{t=1}^{n} EBIT_t (1-T)/n}{(I_0 + I_n)/2} \right]
$$

(15)
Where EBIT is earning before interest and tax as,

\[ T = \text{Tax rate}, \]

\[ I_0 = \text{book value of investment in the beginning} \]

\[ I_n = \text{book value of investment at the end of n number of years}. \]

3. **Net Present Value (NPV)**

NPV is mathematically defined as

\[ \text{NPV} = \sum_{t=1}^{N} \frac{NCF_t}{(1 + k)^t} - I_0 \]  \hfill (16)

Where \( NCF_t \) is the net cash flow in time period

\[ I_0 \] is the initial cash outlay

\[ K \] is the firm’s cost of capital

\[ N \] is the number of years in the project.

4. **Internal Rate of Return (IRR):** It is the rate which equates the present value of the cash flows and inflows.

\[ \text{NPV} = 0 = \sum_{t=1}^{N} \frac{NCF_t}{(1 + \text{IRR})^t} - I_0 \]  \hfill (17)

5. **Profitability Index (PI):** This is the ratio of the present value of the cash flows to the initial outlay. Mathematically we have

\[ \text{PI} = \frac{\text{PV of annual cashflows}}{\text{Initial investment}} \]

\[ \text{PI} = \sum_{t=1}^{N} \frac{NCF_t}{(1 + k)^t} \frac{1}{I_0} \]  \hfill (18)

3. Advanced Capital Budgeting Decision

A. Capital budgeting decision under inflation
Firms are always concerned about the impact of inflation on the project’s profitability. The capital budgeting results will be biased if the impact of inflation is not correctly factored in the analysis. Since the opportunity cost of capital or the discount rate is a combination of the real rate (say K) and the expected inflation rate (call it alpha $\alpha$). This relationship, long ago recognized in the financial economic theory is called the Fisher’s Effect. It may be stated as follows.

Nominal discount rate = (1 + Real discount rate) x (1+ inflation rate) – 1

$$K = (1 + K)(1 + \varepsilon) - 1$$

(19)
The NPV formula can be written as follows when cash flows and discount rates are expressed in normal terms.

$$\text{NPV} = \sum_{t=0}^{n} \frac{NCF_t (1 + \alpha)^t}{(1 + k)^t} - I_0$$

(20)

When $K$ is the real discount rate,

$\alpha$ is the expected inflation rate.

B. Asset Replacement Problem

The choice between projects with unequal lives should be made by comparing their real annual equivalent values (AEVs) AEV is the NPV of an investment divided by the annuity factor given its life and host-free discount rate.

$$\text{AEV} = \frac{NPV}{\text{Annuity factor}}$$

(21)

Where $PVFA = \frac{1}{(1 + K)^n}$

AEV for perpetuities

When we assume that projects can be replicated at constant scale indefinitely, we imply that an annuity is paid at the end of every n years starting from the first period. This can be written as

$$\text{NPV}_\infty = NPV_n \frac{NPV_n}{(1 + k)^n} + \frac{NPV_n}{(1 + k)^{2n}} + ....$$

(22)

Solving this series, we get
\[ NPV_\infty = (NPV_n) \left[ \frac{(1 + k)^n}{(1 + k)^n - 1} \right] \]  

(23)

Where

\( NPV_\infty \) is the present value of the investment indefinitely

\( NPV_n \) is the present value of the investment for original life.

\( n \) and \( k \) is the opportunity cost of capital.

**NOTE:** The procedure of comparing AEVs can be followed while replacing an existing asset by new asset. The NPV rule also proves handy in resolving the timing problem of an investment.

### C. Capital Rationing

Capital rationing occurs because of either the external or internal constrain on the supply of funds. In capital rationing situation, the firm cannot accept all profitable projects. Therefore, the firm will aim at maximizing NPV subject to the funds constraint.

- In simple one-period capital rationing situations, the profitability index (PI) rule can be used. PI rule breaks in the case of multi-period funds constraints and project indivisibility. The PI under capital rationing situation is as

\[
PI = \sum_{t=1}^{n} \frac{C_t}{(1 + K)^t} / I_0
\]  

(24)

i.e. the ratio of the present value of cashflows to the initial outlay.

- A more sophisticated approach either linear programming or integer programming can be used to select investment under capital rationing. However, two factors limit the use of these approaches in practice. First, they are costly and second they assume investment opportunities as known.

### 4. THE THEORY OF CHOICE (UTILITY THEORY)

**A. The Axioms of Cardinal Utility**
Comparability: Here, an individual can say either that outcome X is preferred to outcome Y (X>Y) or Y is preferred to X (Y>X) or the individual is indifferent as to X and Y (X~Y).

Consistency: If an individual prefers X to Y and Y to Z, then X is preferred to Z (if X > Y and Y > Z, then X > Z). If an individual is indifferent as to X and Y and is also indifferent as to Y and Z, then he is indifferent as to X and Z. (if X ~ Y and Y ~ Z, then X ~ Z).

Strong Independence: assume a gamble where an individual has a probability \( \alpha \) of receiving outcome X and a probability (1-\( \alpha \)) of a receiving outcome Z. this can be written a \( \text{G}(X,Z: \alpha) \). The third axioms says that if the individual is indifferent as to X and Y, then he will also be indifferent as to a first gamble, set up between X with probability \( \alpha \) and mutually exclusive outcomes Z, and a second gamble, set up between Y with probability \( \alpha \) and the same mutually exclusive outcome, Z.

If X ~ Y, then \( \text{G}(X, z: \alpha) \sim \text{G}(Y,Z: \alpha) \)

Measurability: If outcome Y is preferred less than X but more than Z, then there is a unique \( \alpha \) (a probability) such that the individual will be indifferent between Y and a gamble X with probability \( \alpha \) and Z with probability 1-\( \alpha \). If \( X>Y> Z \) or \( X > Y > Z \), then there exists a unique \( \alpha \), such that \( Y \sim \text{G}(X,Z: \alpha) \)

Ranking: If alternatives Y and U both lie somewhere between X and Z and we can establish gambles such that an individual is indifferent between Y and a gamble between X (with probability \( \alpha_1 \)) and Z, while he is also indifferent between U and a second gamble, this time between X (with probability \( \alpha_2 \)) and Z, then if \( \alpha_1 \) is greater than \( \alpha_2 \), Y is preferred to U. If \( X>Y>Z \) and \( X>U \), then if \( Y-\text{G}(X, Z: \alpha_1) \) and \( U-\text{G}(X, Z: \alpha_2) \), it follows that if \( \alpha_1 > \alpha_2 \) then \( Y > U \) or if \( \alpha_1 = \alpha_2 \) then \( Y = U \).

However, given the five axioms of rational investor behavior and the additional assumption that all investors always prefer more wealth to less, we can say that investors will always seek to maximize their expected utility of wealth.

In general, we can express the expected utility of wealth as
\[
\text{MAX } E[U(W)] = \sum P_i U(W_i)
\]

Equation (25) is exactly what we mean by the theory of choice.

We have graphed three utility functions with positive marginal utility:

Risk lover  = If $U[E(W)] < E[U(W)]$ we have risk aversion.

Risk neutral = If $U[E(W)] = E[U(W)]$ we have risk neutrality

Risk averter = If $U[E(W)] > E[U(W)]$ we have risk loving

Note: If an individual’s utility function is strictly concave he will be risk averse, if it is liner, he will be risk neutral and if it is convex, he will be a risk lover.
5. **Objects of Choice: Mean Variance Uncertainty**

A. **Measure of location**

The mean is most often used measure of location. It is defined as

\[ E(\bar{X}) = \sum_{i=1}^{N} P_i X_i \]  

Where \( P_i \) is the probability of a random event  
\( X_i \) is the total number of possible events  

The expected or mean return is the expected prices less the current price divided by the current price.  

\[ E(\bar{R}) = \frac{E(\bar{P}) - P_0}{P_0} \]  

B. **Measures of Dispersion**

There are five measures of dispersion which we could use the range, the semi-interquartile range, the variance, the semi-variance and the absolute mean deviation. Each of these has slightly different implications for risk

- The range is defined as the difference between the highest and lowest outcomes.  
- The semi-interquartile range is the difference between the observation of the 78th percentile, \( X_{75} \), and the 25th percentile, \( X_{25} \) divided by 2.  

\[ \text{Semi-interquartile range} = \frac{X_{75} - X_{25}}{2} \]  

- The variance is the statistic most frequently used to measure the dispersion of a distribution and it will be used as a measure of investment risk. It is defined as the expected squared difference from the mean.

\[ VAR(\bar{X}) = E[ (X_i - E(\bar{X}))^2 ] \]  

Recalling the definition of the mean as the sum of the probabilities of events times the value of the events the variance can be rewritten as

\[ VAR(\bar{X}) = \sum_{i=1}^{N} P_i (X_i - E(\bar{X}))^2 \]  

- The standard deviation which is the square root of the variance, is often used to express dispersion and his given as
\[
\sigma(P) = \sqrt{\text{VAR}(P)}
\] (30)

C. Measuring Portfolio Risk and Return

Investors measure the expected utility of choices among risky assets by looking at the mean and variance provided by combination of those assets. For a portfolio manager, the risk and return are the mean and variance of the weighted average of the assets in his portfolio.

The mean return is the expected outcome and is given as

\[
E(R_p) = aE(X) + bE(Y)
\] (31)

The portfolio mean return is the weighted average of returns of individual securities, where the weights are the percentage invested in those securities.

The portfolio variance is the sum of the variance of the individual securities multiplied by the square of their weights plus a third term, which is called the covariance, Cov \((X, Y)\)

\[
\text{VAR}(R_p) = a^2 \text{VAR}(X) + b^2 \text{VAR}(Y) + 2ab \text{Cov}(X, Y)
\] (32)

\[
\text{Cov}(X, Y) = E[(X - E(X))(Y - E(Y))]
\]

D. The Correlation Coefficient

The correlation \(r_{xy}\), between two random variables is defined as the covariance divided by the product of the standard deviations.

\[
V_{x, y} = \frac{\text{Cov}(X, Y)}{\sigma_x \sigma_y}
\] (33)

\[\text{Fig. 12 Correlation Coefficient Graph}\]
This graph shows independent returns of two assets. A situation where the covariance is zero, the correlation between them is also zero.

An opposite situation occurs when the returns are perfectly correlated. Here the returns of the assets fall on a straight line, perfect correlation will result in a correlation coefficient which is equal to 1.

6. Market Equilibrium

Means Variance Uncertainty and Asset Valuation

In order to determine the market price for risk and the appropriate measure of risk for a single asset, the economic model of the Capital Asset Pricing Model (CAPM) is used to solve this problem.

Recall that the slope of the capital market line is

\[
\frac{E(R_m) - R_f}{\sigma_m} = \frac{E(R_i) - E(R_m)}{\frac{\sigma_{im}^2}{\sigma_m^2}}
\]

Equation (34)

Equating this with the slope of the opportunity set we have

\[
\frac{E(R_m) - R_f}{\sigma_m} = \frac{E(R_i) - E(R_m)}{\sigma_{im}^2/\sigma_m^2}
\]

Equation (35)

This relationship can be rearranged to solve for \(E(R_i)\) as follows:

\[
E(R_i) = R_f + \left[ E(R_m) - R_f \right] \beta_i
\]

Equation (36)

Equation 7.3 is known as the capital asset pricing model and it is shown graphically below.
FIG. 13 The Capital Asset Pricing Model

Note: The required rate of return on any asset, $E(R_i)$ in eq. 36 is equal to the risk-free rate of return plus a risk premium. The risk premium is the price of risk multiplied by the quantity of risk. In CAPM, the price of the risk is the slope of the line, the difference between the expected rate of return on the market portfolio and the risk-free rate of return. The quantity of risk is often called beta, $\beta_i$ it is the covariance.

\[ \beta_i = \frac{\sigma_{im}}{\sigma_{m}^2} = \frac{Cov(R_i, R_m)}{VAR(R_m)} \quad (37) \]

Between returns on the risky asset; I, and market portfolio, M divided by the variance of the market portfolio. The risk-free asset has a beta of zero because its covariance with the market of portfolio is zero. The market portfolio has a beta of one because the covariance of the market portfolio with itself is identical to the variance of the market portfolio:

\[ \beta_m = \frac{Cov(R_m, R_m)}{VAR(R_m)} = \frac{VAR(R_m)}{VAR(R_m)} = 1 \quad (38) \]

B. Use of the CAPM for valuation: Single-Period Models, Uncertainty

The CAPM is an extremely useful tool for valuing risky asset. Our risky return, $\bar{R}_j$ is

\[ \bar{R}_j = \frac{\bar{P}_e - P_0}{P_0} \quad (39) \]

Where $\bar{R}_j$ is risky return

$\bar{P}_e$ is risky payoff

$P_0$ is the price we pay today

The CAPM can be used to determine what the current value of the asset, $P_0$ should be.

The CAPM is

\[ E(R_j) = R_f + \lambda Cov(R_j, R_m) \quad (40) \]

Where $\lambda = \frac{E(R_m) - R_f}{VAR(R_m)}$
Note that $\lambda$ can be described as the market price per unit risk. We can equate the expected return Eq. (39) with the expected return in Eq (40):

$$\frac{E(P_c) - P_o}{P_o} = R_f + \lambda \cdot CoV(R_f, R_m)$$

(41)

We can now interpret $P_o$ as the equilibrium price of the risk asset. Rearranging the above expression, we get

$$P_o = \frac{E(P_c)}{1 + R_f + CoV(R_f, R_m)}$$

(42)

Equation (42) is often referred to as the risk-adjusted rate of return valuation formula. The numerator is the expected end of period price for the risky asset and the denominator can be thought of as a discount rate.

The certainty – equivalent valuation formula is given

$$P_o = \frac{E(P_c) - \lambda \cdot CoV(P_c, R_m)}{1 + R_f}$$

(43)

Note: That the risk-adjusted rate of return and the certainty equivalent approaches are equivalent for one-period valuation models.

C. Applications of the CAPM for Corporate Policy

Assuming that the firm has no debt and that there are no corporate taxes. The cost of equity capital for a firm is given directly by the CAPM, which is use to determine the required rate of return on equity and is given as

$$E(R_j) = R_f + \left[E(R_m) - R_f \right] \beta_j$$

(44)

If it is possible to estimate the systematic risk of a company’s equity as well ask the market rate of return then $E(R_j)$ is the required rate of return on equity i.e. the cost of equity for the firm. If we designate the cost of equity as $K_e$ then

$$E(R_j) = K_e$$

(45)
Graphically it is represented as

![Graph 14](image14)

**Fig 14 The Cost of Equity using the CAPM**

The expected rate of return on project K is higher than the cost of equity for the firm. But the project also is riskier than the firm because it has greater systematic risk.

![Graph 15](image15)

**Fig 15 The Capital Market Line with No Risk-Free Rate**

The above graph shows the capital market line with no risk-free rate. Portfolio M is identified by all investors as the market portfolio which has on the efficient set. Portfolio
A and B are both uncorrelated with the market portfolio M and have the same expected return, \( E(R_z) \). However, only one of them, portfolio B lies in the opportunity set.

7. **Theory of Efficient Capital Markets**

Capital markets may be efficient in weak, semi-strong and strong form as defined by Fama (1970, 1976).

**A. Weak-Form Efficiency**

The security prices reflect all past information about the price movements in the weak form of efficiency. It is therefore, not possible for an investor to predict future security by analysis historical prices, and achieve a return better than the stock market index.

In an efficient capital market, there should not exist a significant correlation between the security prices overtime, as share prices behave randomly. Hence the weak form of efficiency is referred to as the *random walk hypothesis*.

**B. Semi-Strong Form Efficiency**

Here security prices reflect all publicly available information. This implies that an investor will not be able to outperform the market by analyzing the existing company-related or other relevant information available in, say the annual accounts, or financial dailies/magazines.

**C. Strong Form of Efficiency**

In the strong form of efficiency, the prices reflect all published and unpublished public and private information. No investor can earn excess returns using information whether publicly available or not.

**D. The Theory of efficient markets**

*Expected Return or “Fair Game” Models* – the fair-game model is based on the behavior of average returns (not on the entire probability distribution. Its mathematical expression is

\[
E_{j,t+1} = \frac{P_{j,t+1} - P_{jt}}{P_{jt}} - \frac{E(P_{j,t+1}/nt) - P_{jt}}{P_{jt}}
\]

\[= \frac{P_{j,t+1} - E(P_{j,t+1}/nt)}{P_{jt}} \quad (46)\]
Where:

\( P_{jt+1} \) = the actual price of security j next period

\( E(P_{jt + 1/nt}) = \) the predicted end-of-period price of security j given the current information structure.

\( E_{jt, t+1} = \) the difference between actual and predicted returns

Note that equation 46 is written in returns form. If we let the one-period return be defined as

\[
    r_{jt, t+1} = \frac{P_{jt, t+1} - P_{jt}}{P_{jt}}
\]  

(47)

Then equation 46 may be rewritten as

\[
    E_{jt, t+1} = r_{jt, t+1} - E(r_{jt, t+1}/nt) \quad \text{and} \quad E(E_{jt, t+1}) = E(r_{jt, t+1} - E(r_{jt, t+1}/nt)) = 0
\]

(48)

Therefore, a fair game means that, on average, across a large number of samples, the expected return on an asset equals its actual return.

\[ \begin{align*}
    \text{The Submartingale or The Martingale Model} \\
    \text{The submartingale is a fair game where tomorrow’s price is expected to be greater than today’s price. Mathematically, the submartingale model is} \\
    E(P_{jt+1/nt}) > P_{jt} \\
    \text{In returns form this implies that expected returns are positive. This may be written as follows:} \\
    \frac{E(P_{jt+1/nt}) - P_{jt}}{P_{jt}} = E(r_{jt, t+1/nt}) > 0 \quad - \quad - \quad - \quad - \quad - \\
    \end{align*} \]  

(50)

\[ \begin{align*}
    \text{The martingale model is also a fair game, however, with a martingale tomorrow’s price is expected to be the same as today’s price. Mathematically, this is} \\
    E(P_{jt, t+1/nt}) = P_{jt} \\
    \text{In returns form, it is written as} \\
    \frac{E(P_{jt+1/nt}) - P_{jt}}{P_{jt}} = E(r_{jt, t+1/nt}) = 0
\]  

(52)

\[ \begin{align*}
    \text{The random walk model} \\
\end{align*} \]
The random walk model says that there is no difference between the distribution of returns conditional on a given information structure and the unconditional distribution of returns.

\[ f(P_1, \ldots, P_n / n_i - 1) = f(P_1, \ldots, P_n / nt - 1) \] (53)

Equation 53 is a random walk in prices while equation 54 is a random walk in returns:

\[ f(r_1 + 1, \ldots, r_n + 1) = f(r_1 + 1, \ldots, r_n + 1 / n_i) \] (54)

Random walks are much stronger conditions than fair games or martingales because they require all the parameter of a distribution (like the mean, variance, skewness and kurtosis) to be same (identical) if returns follow a random walk, then the mean of the underlying distribution does not charge over time, and a fair game will result.

8. Dividend Policy of the Firm

On the relationship between dividend policy and the value of the firm, different theories and models have been advanced as follows.

A. Relevance Theory

The proponents of these theories are Professor J.E. Walter called the Walters model and Myron Gordon with the popular Gordon’s dividend-capitalization model.

Dividend Relevance Assumption

- All-equity firm
- No external financing
- Constant return and cost of capital
- 100 percent payout or retention

Walter’s model to determine the market price per share is given below.

\[ P = \frac{DIV}{K} + r \frac{(EPS - DIV)}{K} \] (55)

Equation 55 shows that the market price per share is the sum of the present value of two sources of income:
The present value of the infinite stream of constant dividends, $\frac{DIV}{K}$ and

The present value of the infinite stream of capital gains, $\left[\frac{r(EPS - DIV)}{K}\right]/K$

This equation (55) can be rewritten as follows:

$$P = \frac{DIV + (r/K)(EPS - DIV)}{K}$$  (56)

**Gordon’s Model**

According to his model, the market value of a share is equal to the present value of an infinite stream of dividends received by shareholders which is given as:

$$P_0 = \frac{DIV_1}{(1 + K)} + \frac{DIV_2}{(1 + K)^2} + \ldots + \frac{DIV_\infty}{(1 + K)^\infty} = \sum_{t=1}^{\infty} \frac{DIV_t}{(1 + K)^t}$$  (57)

**Note:** That the dividend per share is expected to grow when earnings are retained. Hence when we incorporate growth in earnings dividends, resulting from the retained earnings in the model, the present value of a share is determined as follows:

$$P_0 = \sum_{t=1}^{\infty} \frac{DIV(1 + g)^t}{(1 + K)^t}$$  (58)

When equation (58) is solve it becomes

$$P_0 = \frac{DIV_1}{k-g}$$  (59)

Substituting $Eps_1 (1-b)$ or $DIV_1$ and $br$ for $g$, equation 59 can be rewritten as:

$$P_0 = \frac{Eps_1 (1-b)}{k-br}$$  (60)

Equation 60 is particularly useful for studying the effects of dividend policy on the value of the share.

**B. The case of a normal firm where $r = k$**

Under this situation equation (60) may be expressed as follows

$$P_0 = \frac{Eps_1 (1-b)}{k-br} = \frac{rA(1-b)}{k-br}$$  (61)

(Since $EPS = Ra$, $A = assets per share$)
If \( r = k_1 \) then
\[
P_0 = \frac{EPS(1-b)}{k-br} = \frac{rA(1-b)}{k-br} = \frac{EPS}{K} = \frac{rA}{r} = A \tag{62}
\]
Equation (62) shows that regardless of the firm’s earnings per share, EPS, or risk the firm’s value is not affected by dividend policy and is equal to the book value of asset per share.

C. **Case of the declining firm where \( r < k \)**

Equation (62) indicates that, if the retention ratio, \( b \) is zero or payout ratio \((1-b)\) is 100 percent the value of the share is equal to:
\[
P_0 = \frac{rA}{k} (b = 0) \tag{63}
\]
If \( r < k \) then \( \frac{r}{k} < 1 \) and from eq (63) it follows that \( P_0 \) is smaller than the firms investment per share in assets \( A \).

D. **Dividend Irrelevance: Miller-Modigliani (MM) Hypothesis**

MM’s hypothesis of irrelevance is based on the following assumption
- Perfect capital market
- No taxes
- The firm has a fixed investment policy
- No risk

MM’s derive their valuation model as follows
\[
r = \frac{DIV_1 + (P_1 - P_0)}{P_0} \tag{64}
\]
\[
P_0 = \frac{DIV_1 + P_1}{1+r} = \frac{DIV_1 + P_1}{1+K} \tag{65}
\]
Where \( P_0 = \) is the market price per share at time 0
\( P_1 = \) is the market price per share at time 1
\( DIV_1 = \) is the dividend per share at time 1
\( r = \) is the firms rate of retain
\( k = \) is the firms cost of capital

\[
r = k
\]
Since \( r = k \) in the assumed world of certainty and perfect markets. Both sides of equation \( H \) can be multiplied by the number of shares outstanding, \( n \), we obtain the total value of firms if no new financing exist.

\[
V = nP_0 = \frac{n(DIV_1 + P_1)}{(1 + K)}
\]

(66)

Thus, there does not seem to be a consensus on whether dividend matter or not. In practice, a number of factors will have to be considered before deciding about the appropriate dividend policy of the firm.

9. Capital Structure and Cost of Capital

There exist conflicting theories on the relationship between capital structure and the value of a firm.

A. Relevance of Capital Structure

The traditionalists believe that capital structure affects the firm’s value. One earlier version of the view that capital structure is relevant is the **NET INCOME (NI) APPROACH**. Here NI approach classify firm’s into 1 levered firm (equity and debt), 2 unlevered firm (only equity but no debt)

Value of equity = discounted value of net income

\[
VE = \frac{Net\ Income}{Cost\ of\ equity} - \frac{NI}{ke}
\]

(67)

Similarly the value of a firm’s debt is the discounted value of debt-holders interest income.

\[
VD = \frac{Interest}{Cost\ of\ debt} = \frac{INT}{Kd}
\]

(68)

So, therefore the value of firm is the sum of the value of equity and the value of debt

\[
V_f = VE + VD
\]

(69)

The firm’s overall cost of capital is \( \frac{Net\ operating\ income}{Value\ of\ the\ firm} \).
The firm’s overall cost of capital is the weighted average cost of capital (WACC). Alternately it can be calculated as;

\[
WACC = \text{cost of equity} \times \frac{VE}{V_f} + \text{cost of debt} \times \frac{VD}{V_f}
\]

(71)

Rearranging equation (5), we get

\[
WACC = K_C = Ke \times \left(\frac{V_D}{V_f}\right) + Kd \times \frac{V_D}{V_f}
\]

(72)

\[
WACC = K_C = Ke - (Ke - Kd) \frac{V_D}{V_f}
\]

B. IRRELEVANCE OF CAPITAL STRUCTURE: MM HYPOTHESIS WITHOUT TAXES

Modigliani and Miller (MM) argue that, in perfect capital markets without taxes and transaction cost, a firm’s market value and the cost of capital remain invariant to be capital structure changes. The value of the firm depends on the earnings and risk of its assets rather than the way in which assets have been financed. Financing only changes the way in which the net operating income is distributed between equity holders and debt-holders.

MM’s proposition 1: Firms in same risk class.

Value of the debt-equity firm = value of equity firm

\[
V_f = V_1 = V_u = \frac{No}{Ka}
\]

(73)

In the case of debt-equity firm, WACC = k_0 or K_1 thus

\[
K_0 = K_1 = \frac{NOI}{V_f}
\]

(74)

While the unlevered firm’s WACC or K_u we have
\[ Ku = \frac{NOI}{Vu} \]  

(75)

**The MM Hypothesis under Corporate Taxes:**

In reality, corporate income taxes exist and interest paid to debt-holder is treated as a deductible expenses. Thus, interest payable by firms save taxes. This makes debt financing advantageous.

Interest tax shield = Corporate Tax rate x Interest

\[ INTS = T \times INT = T \times K_dD \]  

(76)

10. Valuation of Corporate Securities

A. Contingent Claims Analysis

Contingent claims analysis (CCA) is a techniques to determine the following:

- The price of a security whose payoff depend upon the price of one or more other securities.
- The value of a convertible bond in terms of the price of the underlying stock into which the bond can be converted.
- The value of the flexibility associated with a multi-purpose production facility.

B. Corporate Liabilities as Options

The most fundamental options are

- Calls options
- Puts option
Fig. 16: Calls option

The graph above depicts the value of the call and put options as it depends on the stock price on the expiration date. The value of the call and put options are

\[
C(S, O, X) = \text{Max}(S - X, 0) \quad (77)
\]

\[
P(S, O, X) = \text{Max}(X - S, 0) \quad (78)
\]

C. Value of the Firm on the Maturity

Fig. 18 and 19 depict the value of equity and risky debt as they depend on the value of the firm on the maturity date of the debt. If on the debt’s maturity date, the value of the firm is greater than the promised principal \( V > B \), then the debt will be paid off, \( D = B \) and the equity will be worth \( V - B \). However, if the value of the firm is less than the promised principal, \( V < B \), then the equity will be worthless \( E = 0 \). Thus, on the maturity date of the debt, the value of equity can be represented as

\[
E(V, O, B) = \text{Max}(V - B, 0) \quad (79)
\]

While the value of risky debt is

\[
D(V, O, B) = \text{Min}(V, B) \quad (80)
\]

Equation (80) says that the value of the risky debt on its maturity date, \( T = O \), is the minimum of \( V \) and \( B \).

![Fig. 18: Value of firm on maturity date](image1)

![Fig. 19: Value of firm on maturity date](image2)

But, since the value of the firm is the sum of the value of the equity and the value of the debt which is given as
V = E + D

11. Agency Theory and Analysis

Agency relationship is a contract in which one or more persons (the principals) engage another person (the agent) to take action on behalf of the principal(s) which involves the delegation of some decision-making authority to the agent.

A. The Agency Problem

- Principal-Agent problems emanates from the conflict of interest (Mekling 1976).
- In most cases, Agent (managers) does not always act in the best interest of the shareholder (principal) and this is the agency problem.
- The agent acts in order to fulfill their own goals at the expense of the principal (owners).

B. The Agency Theory

In developing a theory of agency two approaches have been advanced, which are:

- The Positive Theory of Agency
- The Principal – Agent literature

The positive agency literature is generally non mathematical and empirical oriented where as the principal-agent literature is generally mathematical and non-empirically oriented. Both literatures address the contracting problem among self-interested individuals and assume that in any contracting relationship total agency costs are minimized. The principal-agent literature has concentrated more on analysis of the effects of preference and asymmetric information and less on the effects of the technology of contracting and control.

C. Agency Cost

- Jensen and Meckling (1976) defined agency costs as the sum of the out-of-pocket costs of structuring, administering and enforcing contracts (both formal and informal) plus the residual loss.
Agency cost include all costs frequently referred to as contracting costs, transactions costs, moral hazard costs and information costs.

D. Conflicts of Interest and Agency Problem

Basically, two conflict of interest exist, they are;

- The conflict of interest between managers and stock holder in the corporations. This conflicts generate agency problems between managers and residual claimants when risk bearing is separated from management i.e. when ownership is separated from control (Berle and Means 1932). Such agency cost can be reduced by the multitude of control procedures outlined in the accounting and control literature. Fama and Jensen (1983) recommended that imposing restrictions on residual claims is one of the ways of controlling the agency cost.

- The conflict of interest between bondholder and stockholders. Some corporate decisions increase the wealth of stockholders while reducing the wealth of bondholders and in cases where the wealth transfers are large enough, stock prices can rise from decisions that reduce the value of the firm.

Smith and Warner identify four major sources of conflict between bondholder and stockholders to include:

- Dividend payout
- Claim dilution
- Asset substitution
- Under investment

REFERENCES


