Determination of Optimal Salary via Defined Benefit Pension Plan with Early Retirement

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Abstract

The paper seeks to apply the mathematical model formulated by Adaji, M. O. et al (2015) to determine the financial value of the retirement benefit \( S_f \) by applying Smooth Pasting condition. We examined the smooth pasting condition (tangency condition) along the optimal point at the boundary for an American put by considering the gradient, \( \frac{\partial V}{\partial S} \), more closely and found that as long as \( V(S) \) coincides with the straight line, \( K - S \) gradient equals \(-1\). We used Microsoft excel spread sheet facility to perform the individual computation for last 25 years of service for university Senior Lecturers. The optimal salary \( S_f \) was determined by drawing a vertical line from the tangent perpendicularly to the horizontal axis (salary axis). We recommend the application of this model to individuals as well as cooperate organisations so as maximise the expected retirement benefit. Early retirement alternate should be encouraged so that the teeming population of unemployed youths can take up the vacancies created as a result of early retirement.

Keyword: Smooth Pasting

Introduction

The smooth pasting property (condition), states that the value function must be continuously differentiable everywhere, and yields conditions, which uniquely determine the optimal stopping region. Art of smooth pasting includes a heuristic justification for the differentiability of value functions at optimal stopping thresholds. In pure stopping problems, “smoothness” requires (and means) that the value function is once differentiable, and is known as the smooth pasting condition.

Optimal stopping problems are linked to free boundary problems. This connection was discovered by McKean (1965) and it was formulated as a free boundary problem that can be solved, an extra condition is needed. The principle of smooth pasting provides this condition. It was first adopted by Mikhalevich (1958), and was studied in greater depth by Shiryaev (2006).

It was Bensoussan (1984), and later Karatzas (1988), that first used no-arbitrage methods to show that the price of the American put is the solution to an optimal stopping problem. This work followed that of McKean (1965), who was the first to derive a free boundary problem for the ‘discounted’ American call with gain function

\[
\Phi(S) = e^{-rt}(S - K)^+
\]
2.0 Assumptions of the Model

a) The model satisfies smooth pasting condition
b) A member of the plan would retire when he/she maximizes the benefits of retirement among all possible dates (stopping times) to retire.

c) Optimal stopping problem with a value function $V(S_t) = \sup_{\tau \leq T} e^{-\tau r} E_t V_t(K - S_t)$ satisfies geometric Brownian motion, $dS_t = \mu S_t dt + \sigma S_t dW_t$

d) The infinitesimal generator of the (strong) Markov process $S$ is given by $\mathbb{L}_S V = r S \frac{\partial}{\partial S} + \frac{\sigma^2}{2} S^2 \frac{\partial^2}{\partial S^2}$.

e) Standard Markovian arguments suggest that $V$ from assumption (g) solves the following free boundary problem of parabolic type $\mathbb{L}_S V = r V$

2.1 Parameters and Variables of the Model

The following parameters (functions) and variables are used in this research work:

$V = V(S, t)$ is the financial value of retirement benefit in the time interval $0 < t \leq T$;

$S = S_t$ = salary at time, $t$;

$S_f$ = Optimal salary;

$K$ = strike salary;

$S - K$ = Payoff for the call option (American call option for a fixed $K$ and any given salary, $S$) representing an employer’s option

$K - S$ = Payoff for the put option (American put option for a fixed $K$ and any given salary, $S$) representing an employee’s option

$(S - K)^+ = \max_S (S - K, 0)$ assumed to occur at the optimal boundary (call option)

$(K - S)^+ = \min_S (K - S, 0)$ assumed to occur at the optimal boundary (put option)

$V(S_f)$ = Optimal financial value of retirement benefit (is also the same as the optimal retirement benefits) with respect to salary

$t$ = Time (in year) spent with the pension plan

$r$ = the salary growth rate or Accrual rate

$\mu = (r - \delta)$ is the expected return of the salary (asset)

$\delta$ = annual dividend yield $\delta \geq 0$ of the asset (salary) (when $\delta = 0$, then $\mu = r$)

$\sigma$ = The volatility of the salary (also the standard deviation)

$T$ = Worker's expected retirement time (maturity or expiry time) in years

$\tau$ = stopping time

$\tau_f$ = optimal stopping time
\( C, D \) = continuation set and stopping set respectively
\( W_t \) = geometric Brownian process, (Disturbance factor)
\( k_1, k_2 \) = arbitrary constants,
\( w_+, w_- \) = respective positive and negative roots of an auxiliary equation
\( \mathbb{R}^d = d \)-dimensional Euclidean space
\( L_S \) = infinitesimal operator of \( S \)
\( V_\tau \) = value function at stopping time, \( \tau \),
\( \Phi_\tau \) = gain function at stopping time, \( \tau \),
\( E_S \) = expectation with respect to \( S \)
\( (\Omega, \mathcal{F}, \mathcal{P}) \) = probability space
\( \mathcal{F}_\tau \) = filtration

**Definition 1.** Let \( \Omega \) be some space of functions from \([0, \infty]\) into \( \mathbb{R} \). The shift operator \( \theta_t: \Omega \rightarrow \Omega \) defined by
\[
(\theta_t(\omega))(s) = \omega(t + s)
\]
for \( \omega \in \Omega \) (Typically, we regard \( \omega \in \Omega \) as a sample path of some stochastic process.) Suppose that \( S = (S_t)_{t \geq 0} \) is a stochastic process on the probability space \( \Omega, \mathcal{F}, \mathcal{P} \) the following useful results are given without proof.

**Remark:** The Shift Operator
The shift operator is useful in defining the (strong) Markov property.

**The Measure \( P_S \)**
Let \( W = (W_t)_{t \geq 0} \) be a standard Brownian motion under the measure \( P \). Thus each \( W_t \) is a random variable defined on a probability space \( (\Omega, \mathcal{F}, \mathcal{P}) \), and \( W_0 = 0 \) under \( P \). Now define \( S_t = S + W_t \), for all \( 0 \leq t < \infty \). Then \( S_t \) is a random variable on the same probability space. Moreover, we see that \( S_0 = S \) under \( P \).

**Definition 2.** If a process \( S = (S_t)_{t \geq 0} \) is equipped with the filtration\( ((\mathcal{F}_t)_{t \geq 0}) \), with \( \mathcal{F} = \sigma(\bigcup_{t \geq 0} \mathcal{F}_t) \), then \( S \) has the (strong) Markov property if any of the following three equivalent conditions hold:
\[
\begin{align*}
\mathbb{E}_S(f(X_{\tau+h})|\mathcal{F}_\tau) &= \mathbb{E}_X(f(S_{\tau+h})|S_\tau) \quad (1) \\
\mathbb{E}_S(S_{\tau+h}|\mathcal{F}_\tau) &= \mathbb{E}_{S_t}(f(S_h)) \quad (2) \\
\mathbb{E}_X(Y \circ \theta_t|\mathcal{F}_\tau) &= \mathbb{E}_{S_t}(Y) \quad (3)
\end{align*}
\]
for all \( S \), all stopping times \( \tau \), all \( h > 0 \), any bounded Borel-measurable function \( f \), and any (bounded) \( \mathcal{F} \)-measurable random variable \( Y \).
Remark: The (Strong) Markov Property

The future behaviour of a Markov process is not dependent on its past, but only on its current value.

Definition 3

Let \((S_t)_{t\geq 0}\) be an Ito diffusion with stochastic differential equation given by
\[
dS_t = \mu S_t dt + \sigma S_t dW_t,
\]
(4)

Then the infinitesimal generator is given by
\[
\mathbb{L}_S = \sum_{i=1}^n \mu_i(S) \frac{\partial}{\partial s_i} + \frac{1}{2} \sum_{i,j=1}^n \sigma_{ij}(S) \frac{\partial^2}{\partial s_i \partial s_j}
\]
(5)

It is important to note that for \(n = 1\) the above infinitesimal generator of the strong Markov process \(S\) becomes
\[
\mathbb{L}_S = \mu(S) \frac{\partial}{\partial s} + \frac{1}{2} \sigma^2(S) \frac{\partial^2}{\partial s^2}
\]
(6)

which may be written as
\[
\mathbb{L}_S = \mu S \frac{\partial}{\partial s} + \frac{1}{2} \sigma^2 S^2 \frac{\partial^2}{\partial s^2}.
\]
(7)

Remark: The Infinitesimal Generator

The infinitesimal generator enables us to associate a second order partial differential operator with a stochastic process.

Definition 4. A function \(V\) is called superharmonic if
\[
\mathbb{E}_S(V(S_\tau)) \leq V(S)
\]
(8)
for all \(S\), and for every stopping time \(\tau\). If the process \(S\) has the (strong) Markov property and is adequately regular, this is equivalent to saying that the process \((V(S_t))_{t\geq 0}\) is a supermartingale under \(P_s\), for each \(S\).

Remark: Dynkin’s superharmonic characterisation of the value function for Markov processes is captured in the four statements of the following theorem.

Theorem 1. Let
\[
V(S) = \min_{\tau \in T} \mathbb{E}_S(\Phi(S_\tau))
\]
(9)
where \(S = (S_t)_{t\geq 0}\) is a (strong) Markov process, started at \(S\) under \(P_S\), and \(T\) is a set of stopping times. Suppose that the stopping time \(\tau_f \in T\) is optimal, that is
\[
V(S) = \mathbb{E}_S(\Phi(S_{\tau_f})�).
\]

Then (under certain regularity conditions):

i. The value function \(V\) is the smallest superharmonic function which dominates the gain function
\[ \Phi. \]

ii. \[ \tau_D \leq \tau_f \ P_{S-\text{a.s.}} \] where the stopping time \( \tau_D \) is defined by
\[ \tau_D = \inf\{t \geq 0 \mid S_t \in \mathcal{D}\} \tag{10} \]
and where \( \mathcal{C} = \{S \mid V(S) > \Phi(S)\} \) and \( \mathcal{D} = \mathcal{C}^c = \{S \mid V(S) = \Phi(S)\} \)

iii. The stopping time \( \tau_D \) defined in (10) is optimal.

**Proposition 1: Optimum time for Exercising**

There exists \( S_f \) such that early exercise is worthwhile for \( S \leq S_f \), but not for \( S > S_f \).

**Proof**

Let \( \pi = V + S \) be a portfolio. As soon as
\[ V = (K - S)^+ \] the option can be exercised since the amount
\[ \pi = (K - S)^+ + S = K \] at interest rate, \( r \).

For \( V > K - S \) it is not worthwhile since the value of portfolio is
\[ \pi = V + S > (K - S) + S \geq K \] but after exercising it is equal to \( K \)

The value \( S_f \) depends on time, and it is termed the free boundary value. We have
\[ V(S, t) = (K - S)^+, \ S \leq S_f(t) \]
\[ V(S, t) > (K - S) > S_f(t) \]

Since the free-boundary value is not known, it must be determined with the option price. For large \( S \), the put options satisfy the Black – Scholes equation (Merton, 1973).

That is, \( S \gg K \),
\[ V(S, t) \xrightarrow{s \to \infty} 0 \] and
\[ V(S_f(t), t) = K - S_f(t) \]

Additional conditions are required as these are not sufficient.

These are
\[ S \to \frac{\partial V}{\partial S}(S, t) \] is continuous at \( S = S_f(t) \)

Since for \( S < S_f(t) \)
\[ \frac{\partial V}{\partial S}(S, t) = \frac{\partial}{\partial S}(K - S) = -1 \] also \[ \frac{\partial V}{\partial S_f}(S_f(t), t) = \frac{\partial}{\partial S_f}(K - S_f) = -1 \]
(This is smooth pasting condition).

According to Merton (1973), American Put option can be determined by solving
\[ S \leq S_f(t): \quad V(S, t) = (K - S)^+ \]
\[ S > S_f(t): \quad \frac{\partial V}{\partial t} + \frac{1}{2} \sigma^2 S^2 \frac{\partial^2 V}{\partial S^2} + (r - \delta) \frac{\partial V}{\partial S} - rV = 0 \]
with the endpoint condition

\[ V(S, T) = (K - S) \]

and the boundary conditions

\[
\begin{align*}
\lim_{S \to \infty} V(S, t) & = 0 \\
V(S_f(t), t) & = K - S_f(t), \\
\frac{\partial V}{\partial S_f}(S_f(t), t) & = \frac{\partial V}{\partial S_f}(K - S_f(t)) - 1
\end{align*}
\]  \(11\)

Our formulation follows perpetual American put. Perpetual American put has value function that is a function of the stock price, \(V = V(S)\) only and its optimal stopping boundary is a constant function.

2.3 Derivation of the Solution by the Markovian Method

Adaji, et al (2015) formulated a mathematical model to determine the financial value of retirement benefit, \(V(S)\) (free arbitrage price or the option value) for perpetual American put. The study considers one pricing formulation of American options, namely, the optimal stopping formulation as equivalence of a free-boundary problem. The optimal stopping problem on perpetual American put was formulated and its solutions found. The solutions found were analysed systematically by applying matching value condition, smooth pasting condition, asset equilibrium condition and the boundary condition. They used the free-boundary approach to derive the solution. The model was based on continuous arithmetic average salary given by

\[
V(S) = \begin{cases} 
\frac{\sigma^2}{2r} \left( \frac{2r}{\sigma^2 + 1} \right)^{2r} S \left( \frac{K}{1 + \frac{2r}{\sigma^2}} \right) & \text{for } S \geq S_f \\
K - S & \text{for } S \leq S_f
\end{cases}
\]  \(12\)

where \(K - S\) is the payoff in the case of American put option at any given value of salary, \(S\).

The equation (12) is used to determine the financial value of retirement benefit, \(V(S)\) (free arbitrage price or the option value) for perpetual American put. It can only be determined if its optimal value, \(S_f\) is known. \(V(S_f)\) is the value of the pension benefit that is optimal to retire. Our task here (amongst other things) is to use (12) to determine the financial value of the retirement benefit \(S_f\) by applying Smooth Pasting condition amongst other financial formulations.

We would like to examine the smooth pasting condition (tangency condition) along the optimal point at boundary for an American put. At \(S = S_f\), the value of the optimal point of American put is \(K - S_f\). This is termed as the value matching condition:

\[ V(S_f) = K - S_f. \]  \(13\)
Suppose \( S_f \) is a known continuous function, the pricing model becomes a boundary value problem with a time dependent boundary. However, in the American put option model, \( S_f \) is not known in advance. Rather, it must be determined as part of the solution.

To be able to calculate the unknown boundary \( S_f \), we need the smooth pasting condition, and therefore we consider the gradient, \( \frac{\partial V}{\partial S} \) more closely. From our assumption that the model satisfies smooth pasting condition, it then implies that as long as \( V(S) \) coincides with the straight line, \( K - S \) with gradient equals \( -1 \), and at the contact point, we will draw a vertical line from the tangent perpendicular to the horizontal axis to obtain \( S_f \).

**Results**

We show how the values are simulated using Excel package. The data we use in the computation are from Appendix 1.

\[
V(S) = \begin{cases} 
S^{-\frac{2r}{\sigma^2}} \left( \frac{\sigma^2}{2r} \right) \left( \frac{K}{1 + \frac{2r}{\sigma^2}} \right)^{\left(\frac{2r}{\sigma^2} + 1\right)} & \text{for } S \geq S_f \\
K - S & \text{for } S \leq S_f
\end{cases}
\]

We use Microsoft excel spread sheet facility to perform the individual computation for 25 years for University senior lecturers and 10 years for University Professors as shown in Tables 1 to 2. (see Appendices 1 and 2)

This computation is applicable to various cadres of staff and institutions.

Substituting these parameters into the above programme, we have

\[
= POWER(Ai, -D1) \ast (1/D1) \ast POWER(D2/(1 + D1), (D1 + 1))
\]

where \( i = 1, 2, 3, ..., 25 \).

We compute financial values of retirement benefit for Senior lecturers in the last 25 years of service using simulation. For this category, we use

\[
D1 = \frac{2r}{\sigma^2} = \frac{2(2.5)}{1^2} = 5;
\]

\[
D2 = K = 6020349;
\]

\( S_0 = 3091505 \).
Figure 1: Graph of Financial Value, $V(S)$ of Retirement for University Senior Lecturers in the last 25 years of service when $K = 6020349$, $S_0 = 3091505$, $r = 2.5$, $\sigma = 1$
Figure 2: Graph of Financial Value, $V(S)$ of Retirement for University Senior Lecturers in the last 25 years of service when $K = \$549,150.5$, $S_0 = \$309,150.5$, $r = 2.5$, $\sigma = 1$.
Figure 3: Graph of Financial Value, $V(S)$ of Retirement for University Senior Lecturers in the last 25 years of service when $K = 500000$, $S_0 = 3091505$, $r = 2.5$, $\sigma = 1$
Discussion

Art of Smooth Pasting, includes a heuristic justification for the differentiability of value functions at optimal stopping thresholds. We demonstrate this by considering the behaviour of curve near $S_f(\tau)$. From our assumption that the model certifies smooth pasting condition, it then implies that as long as $V(S)$ coincides with the straight line, $K - S$ its gradient equals $-1$, and at the high contact point produces $S_f$ (where $(K - S) = (K - S_f)$) we also have under the following two scenarios

$$\left. \frac{\partial V}{\partial S} \right|_{S=S_f(\tau)} < -1,$$  $$\left. \frac{\partial V}{\partial S} \right|_{S=S_f(\tau)} > -1$$

When $\left. \frac{\partial V}{\partial S} \right|_{S=S_f(\tau)} < -1$, the salary curve $V(S)$ at value of $S$ close to but greater than $S_f$ falls below the intrinsic value line (see the figure 1).

When $\left. \frac{\partial V}{\partial S} \right|_{S=S_f(\tau)} > -1$, is above the intrinsic value line (see figure 3). The value of the American put option at asset salary level close to $S_f(\tau)$ can be varied (increased by choosing a smaller value for $S_f(\tau)$) This explains why both cases do not correspond to the optimal exercise strategy.

In another way, we simply write $V = (K - S_f)$, $\left. \frac{\partial V}{\partial S_f} \right| = -1$

In order to obtain optimality, we vary the value of $(K - S)$ which represents a straight line. We continue until the line touches the curve tangentially as seen in figure 2.

The $(K - S)$ reveals the relation between the strike salary $K$ (assumed to be greater than other salaries over the years). It has a negative slop which intercept the salary axis (horizontal axis at $K$.) The value of financial benefit (the vertical axis represents $V(S)$ and the horizontal axis represents the salary axis).

Figure 1: Graph of Financial Value, $V(S)$ of Retirement for University Senior Lecturers in the last 25 years of service when $K = 6020349$, $S_0 = 3091505$, $r = 2.5$, $\sigma = 1$. The Figure displays the interaction of the salary curve and the straight line with negative slop. The line $(K - S)$ cuts the curve, $V = V(S)$ at two points. This means that we did not get the optimal point. This implies that the $K = 6020349$ is high and needed to be reduced.

Figure 2: Graph of Financial Value, $V(S)$ of Retirement for University Senior Lecturers in the last 25 years of service when $K=45491505$, $S_0 = 3091505$, $r = 2.5$, $\sigma = 1$. It a graph showing salary, $S$ and financial value of retirement benefit $V(S)$ of American put option for fixed strike.
salary, $K=\$5491505$ per annum and the payoff, $K - S$ of American put option for University Senior lecturers in the last 25 years of service. $S_0 = 3091505$, $\frac{2r}{\sigma^2} = 5$. The value of the option meets the payoff function smoothly. The salary curve $V(S)$ touches tangentially the intrinsic line at the point representing the value of the optimal retirement salary. The optimal value of the financial retirement benefit is got by drawing a vertical line from the tangent point to the horizontal life. The vertical line intercepts the salary axis at $S_f = 4591505$ per annum). The corresponding vertical axis $V(S_f) = 100$.

**Figure 3: Graph of Financial Value, $V(S)$ of Retirement for University Senior Lecturers in the last 25 years of service when $K= 500000$, $S_0 = 3091505$, $r = 2.5$, $\sigma = 1$. This graph is showing salary, $S$ and financial value of retirement benefit $V(S)$ of American put option for fixed strike salary, $K=\$500000$ per annum and the payoff, $K - S$ of American put option for University Senior lecturers in the last 25 years of service. $S_0 = 3091505$, $\frac{2r}{\sigma^2} = 5$**

The Figure 3 displays the interaction of the salary curve and the straight line with negative slop. The line $(K - S)$ and the curve, $V = V(S)$ do not have contact at all. This means that we did not get the optimal point. This implies that the $K = 5000000$ is low and needed to be increased.

The differences we observe in Figures 1 and 3 are direct consequences of varying $K - S$. When $K$ was further reduced from $K = 6020349$ to $= 6000000$, we have the result in Fig 2.

**Conclusion**

In the examples, we used Microsoft excel spread sheet facility to perform the individual computation for 25 years for University senior lecturers and 10 years for University Professors as shown in Tables 1 to 3. (see Appendices 1–3)

We assumed that the smooth pasting condition holds, this assumption was used it to find the optimal stopping region, and verified that this solution equals the optimal value function. This is what is referred to as “Art of Smooth Pasting,” which includes a heuristic justification for the differentiability of value functions at optimal stopping thresholds.

We demonstrated this by considering the behaviour of curve near $S_f(\tau)$. From our assumption that the model certifies smooth pasting condition, it then implies that as long as $V(S_t)$ coincides with the straight line, $K - S$ its gradient equals $-1$, and at the high contact point produces $S_f$ (where
\[(K - S) = (K - S_f)\] we also have under the following two scenarios \[\frac{\partial V}{\partial S} \bigg|_{S=S_f(\tau)} < -1,\]

When \[\frac{\partial V}{\partial S} \bigg|_{S=S_f(\tau)} < -1\], the salary curve \(V(S)\) at value of \(S\) close to but greater than \(S_f\) falls below the intrinsic value line (see the figure1).

When \[\frac{\partial V}{\partial S} \bigg|_{S=S_f(\tau)} > -1\], is above the intrinsic value line (see figure3). The value of the American put option at asset salary level close to \(S_f(\tau)\) can be varied (increased by choosing a smaller value for \(S_f(\tau)\)). This explains why both cases do not correspond to the optimal exercise strategy.

In order to obtain optimality, we varied the value of \((K - S)\) which represents a straight line. We continued until the line touched the curve tangentially as seen in figures 2 and 5.

The \((K-S)\) reveals the relation between the strike salary, \(K\) (assumed to be greater than other salaries over the years). It has a negative slope which intercept the salary axis (horizontal axis at \(K\)). The value of financial benefit (the vertical axis represents \(V(S)\) and the horizontal axis represents the salary axis.

The result compares favourably with that of Calvo-Garrido, et al (2013). In the existing model, the regions in which it is optimal to retire before the retirement date, \(T\) were proposed. This model goes beyond mere regions to optimal points for early retirement.

The stopping time, \(\tau_f\) has a probabilistic interpretation. This is because, if one misses the optimal stopping time, then on average, the payoff he/she will receive in the future will not be as much as that received at the optimal exercise time. So intuitively, the optimal stopping time is a time after which the put option will lose value on average.

The model is people-friendly in application as individual employee who does not have a mathematical background can also use with ease. The application can be extended to every employee’s cadre. We also included the formula and the computation of gratuity as Appendices 1 and 2.

We recommend the application of this model to individuals as well as cooperate organisations so as maximise the expected retirement benefit. This can only to achieve if we know the optimal point to retire. We recommend that government at the federal, state and local level make public the salary growth rate as well as the strike salary of every cadre. Early retirement alternative should be
encouraged so that the teeming population of unemployed youths can take up the vacancies created as a result of early retirement.

References


APPENDICES

The computer programme is as follows:

\[ V(S) = \text{POWER}(S, -2r/\sigma^2) \ast \left(1/\frac{2r}{\sigma^2}\right) \ast \text{POWER}\left(\frac{K}{1 + \frac{2r}{\sigma^2}}, \left(\frac{2r}{\sigma^2} + 1\right)\right) \]

We used the first 4 columns, A, B, C, D, E (to represent) as follows:

A = S = salary (horizontal axis)

B = V(S) = financial values of retirement benefit (vertical axis)

C = K - S

D1 = \frac{2r}{\sigma^2}

D2 = K
Substituting these parameters into the above programme, we have

\[ = \text{POWER}(A_i, -D_1) \ast (1/D_1) \ast \text{POWER}(D_2/(1 + D_1), (D_1 + 1)) \]

where \( i = 1, 2, 3, \ldots, 25 \).

**Table 1**

Simulated values of data in Table 3 using for the model equation (3.55) for University Senior lecturers in the last 25 years of service when \( K = 6020349, S_0 = 3091505, \frac{2r}{\sigma^2} = 5 \)

<table>
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<th>S</th>
<th>V=V(S)</th>
<th>V= (K-S)</th>
<th>X= .5</th>
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<td>2928844</td>
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Table 2

Simulated values of data in Table 3 using for the model equation (3.55) for University Senior lecturers, in the last 25 years of service when $K = 5491505, S_0 = 3091505, \frac{2r}{\sigma^2} = 5$

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Available online: [http://internationaljournalofresearch.org/](http://internationaljournalofresearch.org/)
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### Table 3
Recommended Consolidated University Salary Structure (CONUASS)

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<th>CONUASS:</th>
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