COMPLEX INTUITIONISTIC FUZZY SOFT SETS

M. THIRUCHERAN\textsuperscript{a}, M. VINOTHKUMAR\textsuperscript{b}

\textsuperscript{a}Associate Professor –P.G and Research Department of Mathematics, Loganatha Narayanasamy Govt. College Ponneri - 601204, Tamilnadu, India.

\textsuperscript{b}Research Scholar –P.G and Research Department of Mathematics, Loganatha Narayanasamy Govt. College Ponneri - 601204, Tamilnadu, India.

\textsuperscript{b}vinothsiva@rocketmail.com

Abstract: This paper presents a new concept of Complex Intuitionistic Fuzzy Soft Set (CIFSS) which is generalized from the innovative concept of a soft complex fuzzy set by adding the non-membership term to the definition of Soft complex fuzzy set. The novelty of complex intuitionistic soft set lies in its ability for membership and non-membership functions to achieve more range of values. The ranges of values are extended to the unit circle in complex plane for both membership and non-membership functions instead of [0, 1] as in the conventional intuitionistic fuzzy functions. We define basic operations namely complement, union, and intersection on complex intuitionistic fuzzy soft set. Properties of these operations are derived.

Keywords: Fuzzy sets; Intuitionistic fuzzy sets; Complex fuzzy sets; Soft complex fuzzy sets; Complex intuitionistic fuzzy sets; Complex intuitionistic fuzzy soft sets

1. Introduction

The idea of the concept of intuitionistic fuzzy set (IFS) was introduced by Atanassov [2], where he achieved his concept by adding the non-membership term to the definition of fuzzy set (FS) that was given by Zadeh [9], while the fuzzy set has only basic component a membership function. It is well known that the range of each membership and non-membership functions are limited to [0, 1], where it belongs to the real numbers. In 2002, Daniel Ramot et al. [7] introduced a new innovative concept and called it complex fuzzy set (CFS). Molodtsov [6] introduced the Theory of Soft Set and established the fundamental results related to this theory. [1] introduced a new approach that is Complex intuitionistic fuzzy set. Basic operations namely complement, union, and intersection on complex intuitionistic fuzzy set. Properties of these operations are derived in [1]. [8] introduced Soft complex fuzzy set, it is an extension of complex fuzzy set defined in [7].

We can utilize concept of Soft complex fuzzy set to represent the decision making problems with uncertainty, and periodicity simultaneously.

In this paper we define a complex intuitionistic fuzzy soft sets and define its operations of complement, union, intersection. We can also define the values of belongingness and non-belongingness for any decision making problem in these complex-valued functions.

2. Definitions

Definition 2.1: A fuzzy set $A$ in a universe of discourse $U$ is characterized by a membership function $\mu_A(x)$ that takes values in the interval $[0, 1]$. A complex fuzzy set $C$ is characterized by a membership function $\mu_C(x)$ that takes values in the complex plane $\mathbb{C}$.

Definition 2.2: An intuitionistic fuzzy set (IFS) $A$ in a non-empty set $U$ (a universe of discourse) is an object having the form:

$A=\{(x, \mu_A(x), \gamma_A(x)) : x \in U\}$, where the functions $\mu_A(x) : U \rightarrow [0,1]$ and $\gamma_A(x) : U \rightarrow [0,1]$, the degree of membership and degree of non-membership of each element $x \in U$ to the set $A$ respectively, and $0 \leq \mu_A(x) + \gamma_A(x) \leq 1$ for all $x \in U$.

Definition 2.3: Ramot et al. [3] proposed an important extension of these ideas, the Complex Fuzzy Sets, where the membership function $\mu$ instead of being a real valued function with the
E. A pair \((F, A)\) is \(\alpha \subset \bar{\omega} \subset \omega\), respectively, that and \(\alpha\) are both real valued giving the range as the unit circle.

However, this concept is different from fuzzy complex number introduced and discussed by Buckley and Zhang. Essentially as explained in [4] this still retains the characterization of the uncertainty through the amplitude of the grade of membership having a value in the range of \([0,1]\) whilst adding the membership phase captured by fuzzy sets. As explained in Ramot et al [3], the key feature of complex fuzzy sets is the presence of phase and its membership.

**Definition 2.4:** [1] A complex intuitionistic fuzzy set \(S\), defined on a universe of discourse \(U\), is characterized by membership and non-membership functions \(\mu_s(x)\) and \(\gamma_s(x)\), respectively, that assign any element \(x \in U\) a complex-valued grade of both membership and non-membership in \(S\). By definition, the values of \(\mu_s(x)\), \(\gamma_s(x)\), and their sum may receive all lying within the unit circle in the complex plane, and are on the form \(\mu_s(x) = r_s(x)e^{j\omega_s(x)}\) for membership function in \(S\) and \(\gamma_s(x) = k_s(x)e^{j\rho_s(x)}\) for non-membership function in \(S\), where \(j = \sqrt{-1}\), each of \(r_s(x)\) and \(k_s(x)\) are real-valued and both belong to the interval \([0,1]\) such that \(0 \leq r_s(x) + k_s(x) \leq 1\), also \(\omega_s\) and \(\rho_s\) are real valued. We represent the Complex intuitionistic fuzzy set \(S\) as \(S = \{(x, \mu_s(x), \gamma_s(x))\mid x \in U\}\), where \(\mu_s(x): U \to \{\alpha \mid \alpha \in \mathbb{C}, |\alpha| \leq 1\}\), \(\gamma_s(x): U \to \{\alpha' \mid \alpha' \in \mathbb{C}, |\alpha'| \leq 1\}\).

**Definition 2.5:** [8] Let \(U\) be an initial set and \(E\) be the set of parameters. \(C(U)\) denotes complex fuzzy power set of \(U\), and let \(A \subset E\). A pair \((F, A)\) is called a soft complex fuzzy set over \(U\), where \(F\) is a mapping given by \(F: A \to C(U)\) and also

\[
F(x) = \{\{a \mid a \in \mu_s(x), a > x \in A\} | a \leq 1\}.
\]

where \(\mu_s(x): U \to \{a | a \in \mathbb{C}, |a| \leq 1\}\).

**Example 2.6:** [8]
Let \(U = \{H_1(India), H_2(Australia), H_3(UK), H_4(USA)\}\) be an initial set, consider \(E = \{e_t(\text{Inflation rate}), e_2(\text{population growth}), e_3(\text{Unemployment rate}), e_4(\text{share market index})\}\) be an country’s growth parameters set and \(A \subset E, A = \{e_1, e_2\}\), then soft complex fuzzy set \((C, A)\), where \(C: A \to C(U)\), may be represented as \((C, A) = (\alpha \mu, \eta \alpha)\).

\[
F(e_1) = \left\{ \frac{0.4e^{j0.7\pi}}{H_1}, \frac{0.8e^{j0.5\pi}}{H_2}, \frac{0.8e^{-j0.2\pi}}{H_3}, \frac{1.0e^{-j0.5\pi}}{H_4} \right\}.
\]

\[
F(e_2) = \left\{ \frac{0.6e^{j0.7\pi}}{H_1}, \frac{0.9e^{j0.9\pi}}{H_2}, \frac{0.7e^{j0.09\pi}}{H_3}, \frac{0.75e^{j0.05\pi}}{H_4} \right\}.
\]

**Complex Intuitionistic Fuzzy Soft Sets (CIFSS)**

**Definition 3.1.** [2] Let \(\tilde{R}_{\alpha} = (\tilde{R}_{\mu_{\alpha}}, R_{\gamma_{\alpha}})\) be an intuitionistic fuzzy subset of \(X \times Y\), and \(R_{\alpha}\) is defined as intuitionistic fuzzy relationship from \(X\) to \(Y\). We write \(X \to Y\) and \(\tilde{R}(x, y)\) denotes the degree of correspondence between \(x\) and \(y\) based on the relationship \(\tilde{R}\). Let \(F(X \times Y)\) denotes the family of an intuitionistic fuzzy relationship on \(X\) to \(Y\). The set \(\tilde{R}_{\alpha} = \{(x, y) \in X \times Y : \tilde{R}_{\mu_{\alpha}}(x, y) \geq \alpha\}

and \(\tilde{R}_{\gamma_{\alpha}}(x, y) \leq \alpha\) \(\subset X \times Y\) is defined as \(\alpha\)-cut set, if \(\tilde{R} \in F(X \times Y)\) for \(\alpha \in [0,1]\).

**Definition 3.2.** [5] Let \(U\) be an initial set and \(E\) be a set of parameters. \(P(U)\) denotes the power set of \(U\). Let \(A \subset E\). A pair \((F, A)\) is called a soft intuitionistic fuzzy set over \(U\), where \(F\) is a mapping given by \(F: A \to P(U)\) and

\[
F(x) = \{y \in U : (x, y) \in \tilde{R}_{\alpha}, \forall x, y \in U, \forall \alpha \in [0,1]\}
\]

**Definition 3.3.** Let \(\tilde{S}_\alpha = (\tilde{S}_{\mu_{\alpha}}, \tilde{S}_{\eta_{\alpha}})\) be an complex intuitionistic fuzzy subset of \(X \times Y\), and \(\tilde{S}_\alpha\) is defined as complex intuitionistic fuzzy relationship from \(X\) to \(Y\). We write \(X \to Y\), and \(\tilde{s}(x, y)\) denotes the degree of correspondence between \(x\) and \(y\) based on the relationship \(\tilde{s}\). Let \(C(X \times Y)\) denotes the family of an complex intuitionistic fuzzy relationship on \(X\) to \(Y\). The set
\[ \bar{s}_{\alpha} = \left\{ (x, y) \in X \times Y : \left| \bar{s}_{\mu_{\alpha}}(x, y) \right| \geq \alpha \right\} \subseteq X \times Y \]

is defined as \( \alpha \)-cut if \( \bar{s} \in C(X \times Y) \) for \( \alpha \in [0, 1] \).

**Definition 3.4.** Let \( U \) be an initial set, \( E \) be a set of parameters. Let \( C(U) \) denotes the complex fuzzy power set of \( U \). Let \( A \subseteq E \), a pair \((S, A)\) is called an complex intuitionistic fuzzy soft set over \( U \), where \( S \) is a mapping given by \( S : A \rightarrow C(U) \) and also

\[ S(x) = \{ y \in U : (x, y) \in \bar{s}_{\alpha}, x \in A, y \in U, \alpha \in [0, 1] \}. \]

**Example 3.5:**

Let \( U = \{ H_1(\text{India}), H_2(\text{Australia}), H_3(\text{UK}) \} \) be an initial set, consider \( E = \{ e_1(\text{Inflation rate}), e_2(\text{population growth}), e_3(\text{Unemployment rate}), e_4(\text{share market index}) \} \) be an country’s growth parameters set and \( A \subseteq E, A = \{ e_1, e_2 \} \), then soft complex intuitionistic fuzzy set \( (F, A) \), where \( S : A \rightarrow C(U) \).

\[ \bar{s}_{\alpha} = \left\{ (e_1, H_1), (e_1, H_2), (e_1, H_3), (0.6e^{0.7x} - 0.5e^{0.2x}), (0.9e^{0.9x} - 0.1e^{0.4x}), (0.6e^{0.7x} - 0.5e^{0.2x}), (0.9e^{0.9x} - 0.1e^{0.4x}) \right\}. \]

Then, a complex intuitionistic fuzzy soft set represented as,

\[ S(e_1) = \begin{bmatrix} (0.4e^{0.7x} - 0.6e^{0.3x}), (0.8e^{0.9x} - 0.3e^{0.3x}) \hline (e_1, H_1) \end{bmatrix}, \begin{bmatrix} (0.8e^{0.5x} - 0.2e^{0.5x}) \hline (e_1, H_2) \end{bmatrix}, \begin{bmatrix} (0.6e^{0.7x} - 0.5e^{0.2x}) \hline (e_1, H_3) \end{bmatrix}, \begin{bmatrix} (0.7e^{0.9x} - 0.3e^{0.4x}) \hline (e_1, H_4) \end{bmatrix} \]

4. **Basic Operations On Soft Complex Intuitionistic Fuzzy Sets**

We will show in this section that the generalization of soft complex fuzzy set to soft complex intuitionistic fuzzy set explains a need to find out some basic operations on the CIFSS, such as complement, union, and intersection. Each of soft complex intuitionistic, fuzzy complement, union and intersection is defined below.

**Definition 4.1.** For two complex intuitionistic fuzzy soft sets \( (S_1, A) \) and \( (S_2, B) \) over a common universe \( U \), we say that \( (S_1, A) \) is a complex intuitionistic fuzzy soft subset of \( (S_2, B) \) i.e. \( (S_1, A) \subseteq (S_2, B) \), if:

(i) \( A \subseteq B \) iff \( \forall x \in E, |\mu_x(x)| \geq |\mu_{y_1}(x)| \) and \( |\eta_x(x)| \geq |\eta_{y_1}(x)| \)

(ii) \( \forall x \in A, S_1(x) \) is an complex intuitionistic fuzzy subset of \( S_2(x) \),

**Definition 4.2.** Two complex intuitionistic fuzzy soft sets \( (S_1, A) \) and \( (S_2, B) \) over a common universe \( U \) are said to be complex intuitionistic fuzzy soft sets equal if \( (S_1, A) \subseteq (S_2, B) \) and \( (S_2, B) \subseteq (S_1, A) \).

**Definition 4.3.** The complement of complex intuitionistic soft fuzzy set \( (S, A) \) is denoted by \( (S, A)^C \) and defined by \( (S, A)^C = (S^C, \neg A) \) where \( S^C : \neg A \rightarrow C(U) \) is a mapping given by \( S^C(A) = \) complex intuitionistic fuzzy complement of \( S(\neg x), \forall x \in \neg A \).
Definition 4.4. The union of two complex intuitionistic fuzzy soft sets \((S_1, A)\) and \((S_2, B)\) over a common universe \(U\) is the complex intuitionistic fuzzy soft set \((S, C)\) where \(C = A \cup B\) and \(\forall x \in C\)

\[
H(x) = \begin{cases} 
S_1(x) & \text{ if } x \in A - B \\
S_2(x) & \text{ if } x \in B - A \\
S_1(x) \cup S_2(x) & \text{ if } x \in A \cap B 
\end{cases}
\]

We write \((S_1, A) \cup (S_2, B) = (S, C)\)

Definition 4.5. The intersection of two complex intuitionistic fuzzy soft sets \((S_1, A)\) and \((S_2, B)\) over a common universe \(U\) is the complex intuitionistic fuzzy soft set \((S, C)\) where \(C = A \cap B\) and \(\forall x \in C\)

\[
H(x) = \begin{cases} 
S_1(x) & \text{ if } x \in A - B \\
S_2(x) & \text{ if } x \in B - A \\
S_1(x) \cap S_2(x) & \text{ if } x \in A \cap B 
\end{cases}
\]

We write \((S_1, A) \cap (S_2, B) = (S, C)\).

Conclusion

In our work, we have put forward some new concept such as complex intuitionistic fuzzy soft set.

It is hoped that our work will enhance this study in soft complex intuitionistic fuzzy sets.

Finally, we provided an example that demonstrated that this method can be successfully worked. It can be applied to problems of many fields that contain uncertainty and periodicity simultaneously. However, the approach should be more comprehensive in the future to solve the related problems.

References


