A Novel Approach for Diminution of Inter-symbol Interference using Pilot Based Channel Estimation for OFDM in Wireless communication System

Darwani Ram Rastogi  
Department of Electronics and Communication Engineering  
Al-Falah School of Engineering & Technology, Faridabad, Haryana, India  
Email- darwani_rastogi@yahoo.com

Javed Ashraf  
Department of Electronics and Communication Engineering  
Al-Falah School of Engineering & Technology, Faridabad, Haryana, India  
Email- jashraf.imi@gmail.com

Dr. Rajeev Ratan  
Electronics & Communication Engineering Department  
MVN University, Palwal, Haryana, India  
Email – rajeeverara@mvenews.com

Abstract:  
Over the past two decades, the rapid development of wireless communication technology has brought great convenience to people’s lives and work. Orthogonal frequency division multiplexing (OFDM) is a modulation technique which is now used in most new and emerging broadband wired and wireless communication systems because it is an effective solution to inter-symbol interference caused by a dispersive channel. In this paper, pilot-based channel estimation of OFDM system is discussed in detail. Focus has been placed on comb type pilot arrangement scheme with two different ways of pilot placement for reducing the inter-symbol interference (ISI). The simulation results show that the performance becomes better as the increasing order of polynomial for interpolation. It is shown that low pass interpolation perform better than other three interpolation techniques.  

Keywords: Orthogonal frequency division multiplexing (OFDM); Inter-symbol Interference (ISI); Channel estimation; minimum Mean square Estimator (MMSE); Least Square (LS)

1. Introduction:  
Since Guglielmo M. Marconi invented wireless telegraph a century ago, the wireless transmission technology allows people to communicate without any physical connection. In recent decades, there is more rapid development of wireless mobile communications, from the initial 1G mobile communication system. Currently, 4G mobile communication system has been started in the world and begun the trial. Development of mobile communication is shown as following step [1, 2]. After demodulating the subcarrier of signals, the spectra information can be obtained from various quarters of FDM as shown in Figure 1.

2. Orthogonal Frequency Division Multiplexing  
Based on the principle of FDM, subcarrier sets of OFDM uses orthogonal sine or cosine function. The orthogonality of \{\cos n\omega t\} and \{\sin n\omega t\} (n, m = 1, 2, 3, -------) occurs in \(t_0, t_0 + T\) as below [3].

\[
\int_{t_0}^{t_0+T} \cos n\omega t \sin m\omega t dt = \begin{cases} 0 & (n \neq m) \\ \frac{T}{2} & (n = m) \\ \frac{T}{2} & (n = m = 0) \end{cases}
\]

\(T = \frac{2\pi}{\omega}\)  
This function is similar to cosine function.

Accordingly to the theory, let frequencies of N subcarriers are \(f_1 (t), f_2 (t) \cdots \cdots \cdots \cdots f_N (t)\) and \(f_k = f_0 + \frac{k}{TN}\), \(k = 1, 2 \cdots \cdots \cdots N\) where TN is unit cost duration.
The single sub-carrier signal is defined as

\[ f_k = \begin{cases} \cos(2\pi f_k t) & 0 \leq t < T \\ 0 & \text{others} \end{cases} \]

We know from the orthogonality that

\[ \int f_n(t)f_m(t)dt = \begin{cases} T & m = n \\ 0 & m \neq n \end{cases} \]

So sub-carriers are orthogonal each other. The receiver can demodulate signal modulated by orthogonality if signal is strictly synchronized.

The OFDM signal is same as FDM as below

\[ S(t) = \sum_{n=-1}^{N} f_n \cos \omega_n (t) \]

But, the difference is spectrum shown as in Figure 2. From Figure 2, FDM requires the protection of a wide interval because of frequency division required in the receiver. But the received OFDM signal spectrum needs smaller bandwidth, since the spectrum of adjacent sub-channels could be overlapped.

### 3 Pilot Based Channel Estimation for OFDM System

OFDM signals can be demodulated either coherently or differentially coherent manners. The most important advantage of differential demodulation is not requiring channel information, and the receiver is relatively simple. However, compared with coherent demodulation, system performance will be degraded from 3 to 4 dB [4]. Moreover, differential demodulation cannot be applied in multi-level modulation. So the coherent demodulation is preferred to achieve higher data.
rates, spectrum efficiency and good performance. Since coherent demodulation depends on the change of phase and amplitude of carrier signal, an accurate estimation of the channel is needed [5, 6]. In OFDM systems, it is very important to know how to get the best channel estimation.

OFDM system channel estimation method can be divided into two ways, pilot-based channel estimation and blind channel estimation. The pilot channel estimation methods are based on the pilot channel and pilot symbol. However, due to two-dimensional time-frequency structure of OFDM system, pilot symbol assisted modulation (PSAM) is more flexible [7]. PSAM is the method doing channel estimation by using pilot sequence and symbol, which are inserted into some fixed positions of signals sent by transmitter. The pilot symbol sent by transmitter makes spectral efficiency and power utilization lower with the trade-off of quick response to the channel variation. Blind channel estimation is focusing on the correlation between the data sent and received, without knowing the information of the transmitted data. Although it yields higher spectral and power efficiencies by using blind channel estimation, it needs more data to analyze. Hence it is suitable for slow varying channel [8-10]. This paper is concentrated on PSAM. For pilot based channel estimation of OFDM system, following three are required. Firstly, suitable pilot pattern needs to be considered. Secondly, pilot-based channel estimation algorithm with low complexity should be identified. Thirdly, proper demodulation method toward effective channel estimation has to be developed.

4 Channel Estimation Schemes:

4.1 Least Square (LS) Estimator

The impulse response of the multipath channel could be written as below

\[ h (t, \tau) = \sum_{k=0}^{L-1} a_k (t) \delta (t - \tau_k) \]  

(5)

Where \( \tau_k \) is the delay of \( k^{th} \) path, \( a_k(t) \) is the amplitude of \( k^{th} \) path, and \( L \) is the number of subcarriers, respectively. Because \( a_k(t) \) is a generalized stationary narrow band complex Gaussian random process, it is independent for each other from different paths. The advantage of LS algorithm is its simplicity, because no consideration of noise and ICI. So, without using any knowledge of the statistics of the channels, the LS estimators are calculated with very low complexity, but obviously it suffers from a high MSE. LS method, in general, is utilized to get initial channel estimates at the pilot subcarriers, which are then further improved via different methods.

4.2 Minimum Mean Square Error (MMSE) Estimator

The MMSE estimator employs the second-order statistics of the channel conditions to minimize the MSE [11]. Let us denote the error of channel estimation as \( e \)

\[ e = H - \hat{H} \]  

(6)

Where \( H \) is actual channel estimation and \( \hat{H} \) is the raw channel estimation, respectively.

And the MSE of the channel is

\[ E\{ |e|^2 \} = E\{ |H - \hat{H}|^2 \} \]  

(7)

\[ E = \{(H - \hat{H})(H - \hat{H})^{H}\} \]

Where \( E \{ \} \) is the expression.

under the lower \((Eb/N0)\) and MMSE estimator could gain 10-15 dB more of performance than LS [12]. However, because of the required matrix inversions, the computation is very complex when the number of subcarriers of OFDM system increases. Therefore, an important drawback of the MMSE estimator can be the high computational complexity.

4.3 Comb Type Pilot Based Channel Estimation

In comb type pilot-based channel estimation, for each transmitted OFDM symbol, \( N_p \) pilot signals are inserted into subcarriers with same interval in frequency from each other [13].

\[ l = 0 \]

\[ X (K) = X (m N_f A + l) = \]
\[ AX (m N_f A + l) \{ X_p (m) \} \{ \text{Data} \} l = 1, 2 \ldots \ldots , N_f - 1 \]  

Where \( X \) (k) is the information, including pilot and data, of all sub-carriers, \( X_p (m) \) & is the value of \( m^{th} \) subcarrier pilot, and \( N_f \) is the frequency interval of inserted pilot, respectively. If there are \( N \) subcarriers,

\[ N_f = \frac{N}{N_p} \]  

According to the pilot position, frequency response of corresponding sub-channel is calculated in the receiver

\[ \hat{H}_P (m) = \frac{Y_p(mp)}{X_p(mN_f)} \quad m = 0, 1 \ldots \ldots , N_p - 1 \]  

Where \( 'X_p \) and \( 'Y_p \) are output and input of pilot subcarrier, respectively. When the pilot interval is shorter than coherent bandwidth, after the frequency response of pilot sub-channel is estimated, interpolation is used in frequency domain to get the channel estimation. Different interpolation methods will yield different accuracy.

### 4.3.1 Linear Interpolation

Linear interpolation performs better than the piecewise constant interpolation [14]. The channel estimation at the data subcarrier is obtained by estimation of response of two adjacent pilot sub-channels. But the precondition is linearity of transmitted functions of adjacent sub-channels. The linear interpolation is shown as below,

\[ \hat{H}(k) = \hat{H}(mN_f + 1) = \hat{H}_P (m + 1) - \hat{H}_P (m) \]  

\[ m = 0, 1, \ldots \ldots , N_p - 1 \]  

In Equation 10, note that only two pilots are used in linear interpolation for channel estimation. Hence complexity of computation is simple. But performance is not necessarily satisfactory.

### 4.3.2 Second Order Interpolation

Second order interpolation is better than linear interpolation, where the channel estimation at the data subcarrier is calculated by used linear combination of three adjacent pilots [15]. Theoretically, high order interpolation yields better channel estimation because of using more pilots. And the channel estimation will be close to the true channel response. But computation complexity also is increased as increasing of order. The channel estimation of second order interpolation is given by

\[ \hat{H}(k) = \hat{H}(mN_f + 1) = C_1 \hat{H}_P (m - 1 + COHPm + C - 1HPm + 1) \]  

The coefficients are defined as,

\[ c_1 = \frac{\alpha (\alpha - 1)}{2} \]

\[ c_0 = - (\alpha - 1)(\alpha + 1), \quad \alpha = \frac{1}{N_f} \]

\[ c_{-1} = \frac{\alpha (\alpha + 1)}{2} \]  

### 4.3.3 Cubic Spline Interpolation

The cubic spline interpolation method produces a smooth and continuous polynomial fitted to given data points, which is given

\[ \hat{H}(k) = \hat{H}(mN_f + 1) = \alpha_1 \hat{H}_P (m + 1) + \alpha_0 \hat{H}_P (m) \]

\[ m = 0, 1, \ldots \ldots , N_p - 1 \]

\( \hat{H}_P (m) \) is the first order derivative of \( \hat{H}_P (m) \), and

\[ \alpha_1 = \frac{3(N_f - 1)^2}{N_f^2} - \frac{2(N_f - 1)^3}{N_f^3} \]

\[ \alpha_0 = \frac{3l^2}{N_f^2} - \frac{2l^3}{N_f^3} \]  

Although cubic spline interpolation with higher order interpolation can be used for better interpolation accuracy, the performance improvement is not obviously proven.
4.3.4 Low Pass Interpolation
The low pass interpolation is performed by inserting zeros into the original sequence and then applying a low pass FIR filter that allows the original data to pass through unchanged and interpolates between such that the mean square error between the interpolated points and their ideal values is minimized.

4.4 Pilot Updation Schemes
There are two ways of pilot updation. (a) Fixed Pilot Updation (b) Dynamic pilot updation

In fixed pilot updation scheme pilot location are kept fixed. It doesn’t change for all those OFDM symbols in which pilots are placed. While in dynamic pilot updation scheme location of pilots in those OFDM symbols in which pilots are placed changes according to the channel coefficients values. In dynamic pilot updation channel is tracked symbol by symbol and pilots are placed at those locations at which fading is high. This scheme gives better result than fixed pilot updation scheme.

5. Simulation Results
This section discusses the simulation setup and results of pilot-based channel estimation in OFDM system. All algorithms of pilot-based channel estimation introduced are simulated by MATLAB. The performance of comb type pilot is analyzed. Different interpolation schemes are compared. Finally a simulation is done for dynamic pilot updation in which instead of inserting pilots at fixed locations for those OFDM symbol in which pilots are placed, pilot locations are updated corresponding to estimated channel coefficients

5.1 Simulation Scenarios
Table 1 shows the default OFDM system parameters. For some simulations of comparison, change of the scenario will be marked if needed.

<table>
<thead>
<tr>
<th>PARAMETERS</th>
<th>SPECIFICATION</th>
</tr>
</thead>
<tbody>
<tr>
<td>FFT Size</td>
<td>256</td>
</tr>
<tr>
<td>Number of Carriers</td>
<td>256</td>
</tr>
<tr>
<td>Pilot ratio</td>
<td>1/8</td>
</tr>
<tr>
<td>Guard length</td>
<td>16</td>
</tr>
<tr>
<td>Guard type</td>
<td>Cyclic Extension</td>
</tr>
<tr>
<td>Modulation Scheme</td>
<td>QPSK / 16 QAM</td>
</tr>
<tr>
<td>Channel</td>
<td>Raleigh Fading</td>
</tr>
<tr>
<td></td>
<td>3-PATH, 5-PATH</td>
</tr>
</tbody>
</table>

5.2 Simulation Results
Simulation Results are arranged in four parts. In the first part channel coefficient plots are shown in which different interpolation techniques linear, square, cubic and low pass are compared for two different channel estimation techniques Least Square (LS) Estimate and Minimum Mean Square Error (MMSE) Estimate. Channel coefficients are measured for two different multipath channels 3 PATH and 5 PATH. Modulation technique used in the simulation is QPSK. SNR is taken 5dB. In the second part scatter plots are shown for different multipath channels, different modulation schemes and for different SNRs. These scatter plots show the in phase and quadrature phase component of the transmitted/received symbols. In the third part Mean Square Error plots are shown which represent comparison between different interpolation techniques, different modulation techniques and different channel estimation techniques. In the fourth part a mean square error plot is shown which compares two different types of pilot updating schemes for Minimum mean square estimator.

5.2.1 Channel coefficients plot:
In the figure 3 channel coefficient plots is shown in which four different interpolation techniques are compared for Least Square Estimate with 3 PATH channel. In this figure 3 first sub plot shows plot of actual channel coefficients and other four subplots are for linear, second order, cubic and low pass interpolation techniques respectively. It is clear from the figure 3 that for low pass interpolation technique channel coefficients are close to the actual channel coefficients. These channel coefficients are measured with least square estimator and for 3 PATH channel.

![Figure 3: Comparison of estimated channel coefficients for different interpolation techniques for Least Square Estimator (3 PATH channel)](image)

In the figure 4 channel coefficient plots is shown in which four different interpolation techniques are compared for Least Square Estimate with 5 PATH channel. In this figure 5 first sub plot shows plot of actual channel coefficients and other four subplots are for linear, second order, cubic and low pass interpolation techniques respectively. It is clear from the figure 5 that for low pass interpolation

![Figure 4: Comparison of estimated channel coefficients for different interpolation techniques for Least Square Estimator (5 PATH channel)](image)
technique channel coefficients are close to the actual channel coefficients. These channel coefficients are measured with least square estimator and with 5 PATH channel. Here we can see that in case of 5 PATH channel also low pass interpolation perform better than other interpolation techniques.
low pass interpolation technique channel coefficients are close to the actual channel coefficients. These channel coefficients are measured with Minimum Mean Square Estimate and with 5 PATH channel. Here we can see that in case of 5 PATH channel also low pass interpolation perform better than other interpolation techniques. From the above figures 5 and figure 6 it is clear that MMSE gives better channel estimation.

5.2.2 Mean Square Error Plots

In figure 7, MSE vs. SNR plots are drawn for fixed pilot updation scheme. In fixed pilot updation scheme pilot locations are kept fixed for each OFDM symbol. In this arrangement scheme pilot locations are independent of channel coefficient values. Figure 7 shows a plot between MSE and SNR. It shows a comparison of different interpolation schemes for MMSE estimate. For this simulation modulation scheme used is QPSK, channel is 3 PATH. From the figure 7, it can be concluded that low pass interpolation technique perform better than other three interpolation techniques and with the continuous tracking of channel and with the up-dation of the pilot locations according to the channel coefficient the mean square error is reduced.

Figure 7: Comparison of different interpolation schemes with QPSK interpolation:

6. Conclusion

In this paper, pilot-based channel estimation of OFDM system is discussed in detail. Focus has been placed on comb type pilot arrangement scheme with two different ways of pilot placement. One is fixed pilot arrangement and second is dynamic pilot arrangement. In fixed pilot arrangement pilot locations are not updated but in dynamic pilot arrangement pilot locations are updated according to channel coefficient values. LS estimator yields the worst performance but with the simplest complexity. Based on LS algorithm, MMSE is analyzed and it shows better performance compared with LS but with more computation complexity. Linear interpolation, second order interpolation, cubic spline interpolation and low pass interpolation are discussed. The simulation results show that the performance becomes better as the increasing order of polynomial for interpolation. It is shown that low pass interpolation perform better than other three interpolation techniques.

References

[1] Taewon Hwang, Chenyang Yang, Gang Wu, Shaoqian Li, Geoffrey Ye Li, —OFDM and Its Wireless Applications: A Surveyl,


